Multiple-Segment Trapezoidal Rule

In Example 1, the true error using a single segment trapezoidal rule was large. We can divide the interval $[8,30]$ into $[8,19]$ and $[19,30]$ intervals and apply the trapezoidal rule over each segment.

$$f(t) = 2000\ln\left(\frac{140000}{140000 - 2100t}\right) - 9.8t$$

$$\int_8^{30} f(t)dt = \int_8^{19} f(t)dt + \int_{19}^{30} f(t)dt$$

$$\approx (19-8)\left[\frac{f(8)+f(19)}{2}\right] + (30-19)\left[\frac{f(19)+f(30)}{2}\right]$$

$$f(8) = 177.27 \text{ m/s}$$

$$f(19) = 2000\ln\left(\frac{140000}{140000-2100(19)}\right) - 9.8(19) = 484.75$$

m/s

$$f(30) = 901.67 \text{ m/s}$$

Hence

$$\int_8^{30} f(t)dt \approx (19-8)\left[\frac{177.27+484.75}{2}\right] + (30-19)\left[\frac{484.75+901.67}{2}\right]$$

$$= 11266 \text{ m}$$

The true error, E_t is

$$E_t = 11061 - 11266$$

$$= -205 \text{ m}$$

The true error now is reduced from 807 m to 205 m. Extending this procedure to dividing $[a,b]$ into n equal segments and applying the trapezoidal rule over each segment, the sum of the results obtained for each segment is the approximate value of the integral.

Divide $(b-a)$ into n equal segments as shown in Figure 4. Then the width of each segment is

$$h = \frac{b-a}{n} \qquad (26)$$

The integral I can be broken into h integrals as

$$I = \int_a^b f(x)dx$$

$$= \int_a^{a+h} f(x)dx + \int_{a+h}^{a+2h} f(x)dx + \ldots + \int_{a+(n-2)h}^{a+(n-1)h} f(x)dx + \int_{a+(n-1)h}^{b} f(x)dx \quad (27)$$

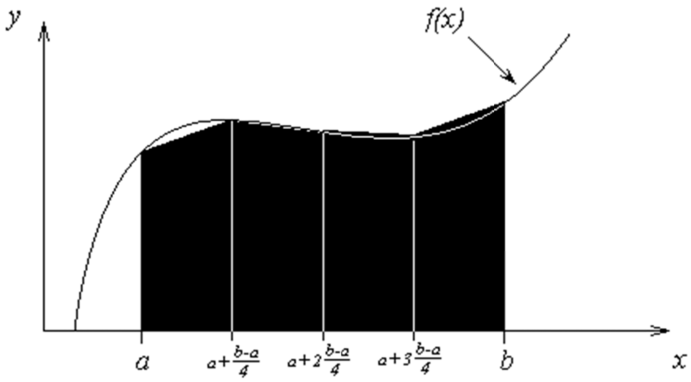

Figure 4 Multiple ($n = 4$) segment trapezoidal rule

Applying trapezoidal rule Equation (27) on each segment gives

$$\int_a^b f(x)dx = [(a+h)-a]\left[\frac{f(a)+f(a+h)}{2}\right]$$

$$+[(a+2h)-(a+h)]\left[\frac{f(a+h)+f(a+2h)}{2}\right]$$

$$+ \ldots\ldots\ldots\ldots\ldots$$

$$+[(a+(n-1)h)-(a+(n-2)h)]\left[\frac{f(a+(n-2)h)+f(a+(n-1)h)}{2}\right]$$

$$+[b-(a+(n-1)h)]\left[\frac{f(a+(n-1)h)+f(b)}{2}\right]$$

$$= h\left[\frac{f(a)+f(a+h)}{2}\right] + h\left[\frac{f(a+h)+f(a+2h)}{2}\right]$$

$$+ \ldots\ldots\ldots\ldots\ldots$$

$$+ h\left[\frac{f(a+(n-2)h)+f(a+(n-1)h)}{2}\right]$$

$$+ h\left[\frac{f(a+(n-1)h)+f(b)}{2}\right]$$

$$= h\left[\frac{f(a)+2f(a+h)+2f(a+2h)+\ldots+2f(a+(n-1)h)+f(b)}{2}\right]$$

$$= \frac{h}{2}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right]$$

$$= \frac{b-a}{2n}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right] \quad (28)$$