Example 1

The vertical distance covered by a rocket from $t = 8$ to $t = 30$ seconds is given by

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use the single segment trapezoidal rule to find the distance covered for $t = 8$ to $t = 30$ seconds.

b) Find the true error, E_t for part (a).

c) Find the absolute relative true error for part (a).

Solution

a) $I \approx (b-a)\left[\frac{f(a) + f(b)}{2} \right]$, where

$a = 8$

$b = 30$

$$f(t) = 2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t$$

$$f(8) = 2000 \ln \left[\frac{140000}{140000 - 2100(8)} \right] - 9.8(8)$$

$$= 177.27 \text{ m/s}$$

$$f(30) = 2000 \ln \left[\frac{140000}{140000 - 2100(30)} \right] - 9.8(30)$$

$$= 901.67 \text{ m/s}$$

$$I \approx (30-8)\left[\frac{177.27 + 901.67}{2} \right]$$

$$= 11868 \text{ m}$$

b) The exact value of the above integral is

$$x = \int_8^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

$$= 11061 \text{ m}$$

so the true error is

$E_t =$ True Value – Approximate Value

$$= 11061 - 11868$$

$$= -807 \text{ m}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$|\epsilon_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100$$

$$= \left| \frac{11061 - 11868}{11061} \right| \times 100$$

$$= 7.2958\%$$