

Conversion of Limits of Integration

In handbooks (see Table 1), coefficients and arguments given for n -point Gauss quadrature rule are given for integrals of the form

$$\int_{-1}^1 g(x) dx \approx \sum_{i=1}^n c_i g(x_i) \quad (19)$$

Table 1 Weighting factors c and function arguments x used in Gauss quadrature formulas

Points	Weighting Factors	Function Arguments
2	$c_1 = 1.000000000$	$x_1 = -0.577350269$
	$c_2 = 1.000000000$	$x_2 = 0.577350269$
3	$c_1 = 0.555555556$	$x_1 = -0.774596669$
	$c_2 = 0.888888889$	$x_2 = 0.000000000$
	$c_3 = 0.555555556$	$x_3 = 0.774596669$
4	$c_1 = 0.347854845$	$x_1 = -0.861136312$
	$c_2 = 0.652145155$	$x_2 = -0.339981044$
	$c_3 = 0.652145155$	$x_3 = 0.339981044$
	$c_4 = 0.347854845$	$x_4 = 0.861136312$
5	$c_1 = 0.236926885$	$x_1 = -0.906179846$
	$c_2 = 0.478628670$	$x_2 = -0.538469310$
	$c_3 = 0.568888889$	$x_3 = 0.000000000$
	$c_4 = 0.478628670$	$x_4 = 0.538469310$
	$c_5 = 0.236926885$	$x_5 = 0.906179846$
6	$c_1 = 0.171324492$	$x_1 = -0.932469514$
	$c_2 = 0.360761573$	$x_2 = -0.661209386$
	$c_3 = 0.467913935$	$x_3 = -0.238619186$
	$c_4 = 0.467913935$	$x_4 = 0.238619186$
	$c_5 = 0.360761573$	$x_5 = 0.661209386$
	$c_6 = 0.171324492$	$x_6 = 0.932469514$

So if the table is given for $\int_{-1}^1 g(x)dx$ integrals, how does one

solve $\int_a^b f(x)dx$?

The answer lies in that any integral with limits of $[a, b]$ can be converted into an integral with limits $[-1, 1]$. Let

$$x = mt + c \quad (20)$$

If $x = a$, then $t = -1$

If $x = b$, then $t = +1$

such that

$$\begin{aligned} a &= m(-1) + c \\ b &= m(1) + c \end{aligned} \quad (21)$$

Solving the two Equations (21) simultaneously gives

$$\begin{aligned} m &= \frac{b-a}{2} \\ c &= \frac{b+a}{2} \end{aligned} \quad (22)$$

Hence

$$\begin{aligned} x &= \frac{b-a}{2}t + \frac{b+a}{2} \\ dx &= \frac{b-a}{2}dt \end{aligned}$$

Substituting our values of x and dx into the integral gives us

$$\int_a^b f(x)dx = \int_{-1}^1 f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right) \frac{b-a}{2}dx \quad (23)$$