

Trapezoidal Rule for Discrete Functions with Unequal Segments

For a general case of a function given at n data points $(x_1, f(x_1))$, $(x_2, f(x_2))$, $(x_3, f(x_3))$, ..., $(x_n, f(x_n))$, where, x_1, x_2, \dots, x_n are in an ascending order, the approximate value of the integral

$\int_{x_1}^{x_n} f(x) dx$ is given by

$$\begin{aligned} \int_{x_1}^{x_n} f(x) dx &= \int_{x_1}^{x_2} f(x) dx + \int_{x_2}^{x_3} f(x) dx + \dots + \int_{x_{n-1}}^{x_n} f(x) dx \\ &\approx (x_2 - x_1) \frac{f(x_1) + f(x_2)}{2} + (x_3 - x_2) \frac{f(x_2) + f(x_3)}{2} + \dots \\ &\quad \dots + (x_n - x_{n-1}) \frac{f(x_{n-1}) + f(x_n)}{2} \end{aligned}$$

This approach uses the trapezoidal rule in the intervals $[x_1, x_2]$, $[x_2, x_3]$, ..., $[x_{n-1}, x_n]$ and then adds the obtained values.