Example 2

What is the absolute relative true error for each of the four methods used in Example 1 if the data in Table 1 was actually obtained from the velocity profile of

$$v(t) = \left(2000 \ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right),$$

where v is given in m/s and t in s.

Solution

The distance covered between $t = 11$ and $t = 16$ is

$$s = \int_{11}^{16} \left(2000 \ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$
$$= 1604.9 \ m$$

Method 1

The approximate value obtained using average velocity method was 1845.3 m. Hence, the absolute relative true error, $|\in_t|$, is

$$|\in_t| = \left| \frac{1604.9 - 1845.3}{1604.9} \right| \times 100\%$$
$$= 14.976\%$$

Method 2:

The approximate value obtained using the trapezoidal rule was 1612.2 m. Hence, the absolute relative true error, $|\in_t|$, is

$$|\in_t| = \left| \frac{1604.9 - 1612.2}{1604.9} \right| \times 100\%$$
$$= 0.451\%$$

Method 3:

The approximate value obtained using the direct polynomial was 1604.3 m. Hence, the absolute relative true error, $|\in_t|$, is

$$|\in_t| = \left| \frac{1604.9 - 1604.3}{1604.9} \right| \times 100\%$$
$$= 0.037\%$$

Method 4:

The approximate value obtained using the spline interpolation was 1595.9 m, hence, the absolute relative true error, $|\in_t|$, is

$$|\in_t| = \left| \frac{1604.9 - 1595.9}{1604.9} \right| \times 100\%$$
$$= 0.564\%$$

Table 2 Comparison of discrete function methods of numerical integration

Method	Approximate Value	$\lvert\in_t\rvert$
Average Velocity	1845.3	14.976%
Trapezoidal Rule	1612.2	0.451%
Polynomial Interpolation	1604.3	0.037%
Spline Interpolation	1595.9	0.564%