Example 1

The upward velocity of a rocket is given as a function of time in Table 2 (Figure 5).

Table 2 Velocity as a function of time.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

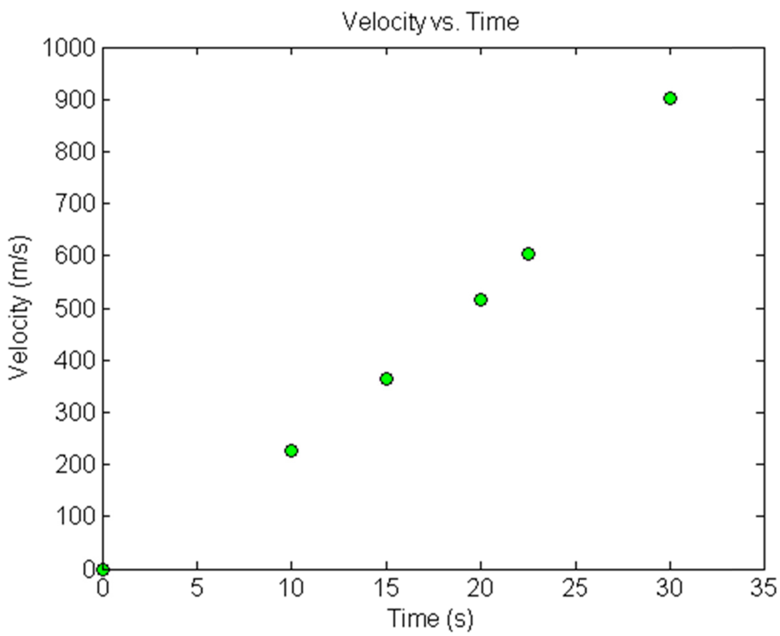

Figure 5 Graph of velocity vs. time data for the rocket example.

Determine the value of the velocity at $t=16$ seconds using linear splines.

Solution

Since we want to evaluate the velocity at $t=16$, and we are using linear splines, we need to choose the two data points closest to $t=16$ that also bracket $t=16$ to evaluate it. The two points are $t_0=15$ and $t_1=20$.

Then

$$t_0=15,\ v(t_0)=362.78$$
$$t_1=20,\ v(t_1)=517.35$$

gives

$$\begin{aligned} v(t) &= v(t_0) + \frac{v(t_1) - v(t_0)}{t_1 - t_0}(t - t_0) \\ &= 362.78 + \frac{517.35 - 362.78}{20 - 15}(t - 15) \\ &= 362.78 + 30.913(t - 15),\ 15 \le t \le 20 \end{aligned}$$

At $t = 16$,

$$\begin{aligned} v(16) &= 362.78 + 30.913(16 - 15) \\ &= 393.7\ \text{m/s} \end{aligned}$$

Linear spline interpolation is no different from linear polynomial interpolation. Linear splines still use data only from the two consecutive data points. Also at the interior points of the data, the slope changes abruptly. This means that the first derivative is not continuous at these points. So how do we improve on this? We can do so by using quadratic splines.