Direct Method - Quadratic

Example 2

The upward velocity of a rocket is given as a function of time in Table 2.

Table 2 Velocity as a function of time.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Determine the value of the velocity at $t = 16$ seconds using the direct method of interpolation and a second order polynomial.

Solution

For second order polynomial interpolation (also called quadratic interpolation), the velocity is given by

$$v(t) = a_0 + a_1 t + a_2 t^2$$

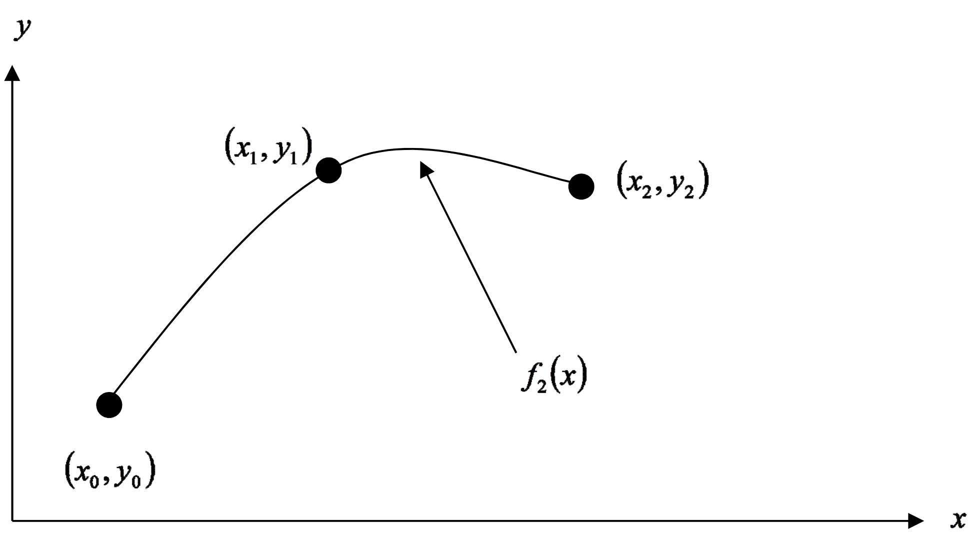

Figure 4 Quadratic interpolation.

Since we want to find the velocity at $t = 16$, and we are using a second order polynomial, we need to choose the three data points that are closest to $t = 16$ that also bracket $t = 16$ to evaluate it. The three points are $t_0 = 10,\ t_1 = 15,$ and $t_2 = 20$.

Then

$$t_0 = 10,\ \ v(t_0) = 227.04$$
$$t_1 = 15,\ \ v(t_1) = 362.78$$
$$t_2 = 20,\ \ v(t_2) = 517.35$$

gives

$$v(10) = a_0 + a_1(10) + a_2(10)^2 = 227.04$$
$$v(15) = a_0 + a_1(15) + a_2(15)^2 = 362.78$$
$$v(20) = a_0 + a_1(20) + a_2(20)^2 = 517.35$$

Writing the three equations in matrix form, we have

$$\begin{bmatrix} 1 & 10 & 100 \\ 1 & 15 & 225 \\ 1 & 20 & 400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$$

Solving the above three equations gives

$$a_0 = 12.05$$
$$a_1 = 17.733$$
$$a_2 = 0.3766$$

Hence

$$v(t) = 12.05 + 17.733t + 0.3766t^2,\ \ 10 \le t \le 20$$

At $t = 16$,

$$v(16) = 12.05 + 17.733(16) + 0.3766(16)^2$$
$$= 392.19 \text{ m/s}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left| \frac{392.19 - 393.70}{392.19} \right| \times 100$$
$$= 0.38410\%$$