Direct Method – Linear

The direct method of interpolation is based on the following premise. Given $n+1$ data points, fit a polynomial of order n as given below

$$y = a_0 + a_1 x + \ldots\ldots\ldots\ldots + a_n x^n \qquad (1)$$

through the data, where $a_0, a_1, \ldots\ldots\ldots, a_n$ are $n+1$ real constants. Since $n+1$ values of y are given at $n+1$ values of x, one can write $n+1$ equations. Then the $n+1$ constants, $a_0, a_1, \ldots\ldots\ldots, a_n$ can be found by solving the $n+1$ simultaneous linear equations. To find the value of y at a given value of x, simply substitute the value of x in Equation 1.

But, it is not necessary to use all the data points. How does one then choose the order of the polynomial and what data points to use? This concept and the direct method of interpolation are best illustrated using examples.

Example 1

The upward velocity of a rocket is given as a function of time in Table 1.

Table 1 Velocity as a function of time.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

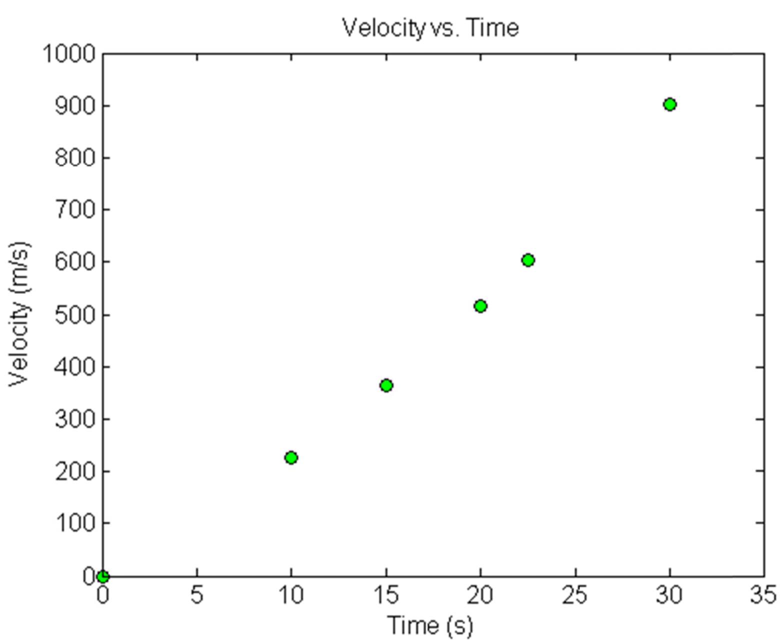

Figure 2 Graph of velocity vs. time data for the rocket example.

Determine the value of the velocity at $t = 16$ seconds using the direct method of interpolation and a first order polynomial.

Solution

For first order polynomial interpolation (also called linear interpolation), the velocity given by

$$v(t) = a_0 + a_1 t$$

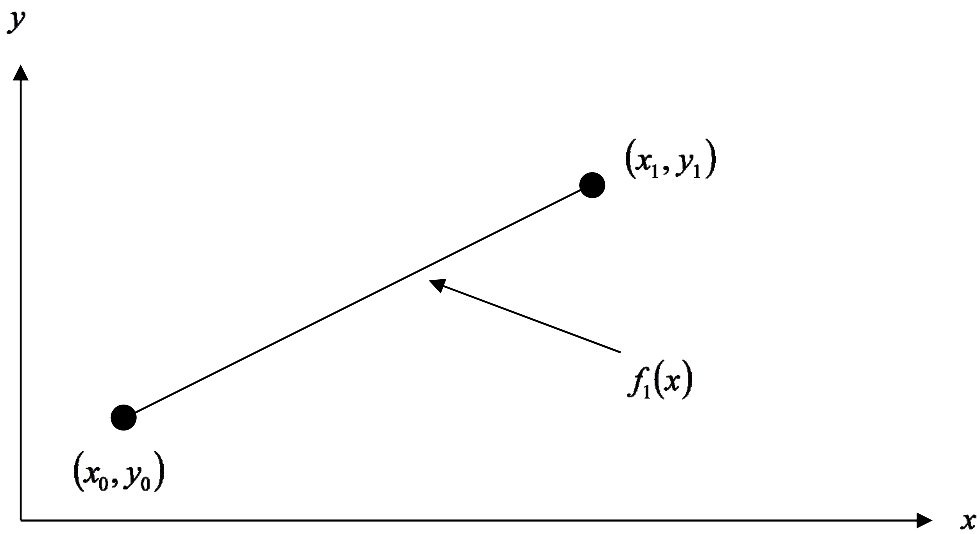

Figure 3 Linear interpolation.

Since we want to find the velocity at $t = 16$, and we are using a first order polynomial, we need to choose the two data points that are closest to $t = 16$ that also bracket $t = 16$ to evaluate it. The two points are $t_0 = 15$ and $t_1 = 20$.

Then

$$t_0 = 15, \; v(t_0) = 362.78$$
$$t_1 = 20, \; v(t_1) = 517.35$$

gives

$$v(15) = a_0 + a_1(15) = 362.78$$
$$v(20) = a_0 + a_1(20) = 517.35$$

Writing the equations in matrix form, we have

$$\begin{bmatrix} 1 & 15 \\ 1 & 20 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \end{bmatrix}$$

Solving the above two equations gives

$$a_0 = -100.93$$
$$a_1 = 30.914$$

Hence

$$\begin{aligned} v(t) &= a_0 + a_1 t \\ &= -100.93 + 30.914t, \; 15 \le t \le 20 \end{aligned}$$

At $t = 16$,

$$\begin{aligned} v(16) &= -100.92 + 30.914 \times 16 \\ &= 393.7 \text{ m/s} \end{aligned}$$