1. <u>Divergence at inflection points</u>

If the selection of the initial guess or an iterated value of the root turns out to be close to the inflection point (see the definition in the appendix of this chapter) of the function $f(x)$ in the equation $f(x)=0$, Newton-Raphson method may start diverging away from the root. It may then start converging back to the root. For example, to find the root of the equation

$$f(x)=(x-1)^3+0.512=0$$

the Newton-Raphson method reduces to

$$x_{i+1}=x_i-\frac{(x_i^3-1)^3+0.512}{3(x_i-1)^2}$$

Starting with an initial guess of $x_0=5.0$, Table 1 shows the iterated values of the root of the equation. As you can observe, the root starts to diverge at Iteration 6 because the previous estimate of 0.92589 is close to the inflection point of $x=1$ (the value of $f'(x)$ is zero at the inflection point). Eventually, after 12 more iterations the root converges to the exact value of $x=0.2$.

Table 1 Divergence near inflection point.

Iteration Number	x_i
0	5.0000
1	3.6560
2	2.7465
3	2.1084
4	1.6000
5	0.92589
6	–30.119
7	–19.746
8	–12.831
9	–8.2217
10	–5.1498
11	–3.1044
12	–1.7464
13	–0.85356
14	–0.28538
15	0.039784
16	0.17475
17	0.19924
18	0.2

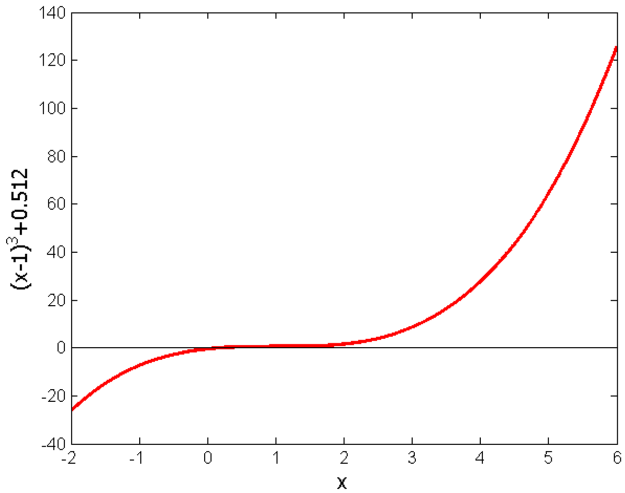

Figure 3 Divergence at inflection point for $f(x) = (x-1)^3 = 0$.