

## Derivation

The Newton-Raphson method is based on the principle that if the initial guess of the root of  $f(x) = 0$  is at  $x_i$ , then if one draws the tangent to the curve at  $f(x_i)$ , the point  $x_{i+1}$  where the tangent crosses the  $x$ -axis is an improved estimate of the root (Figure 1).

Using the definition of the slope of a function, at  $x = x_i$

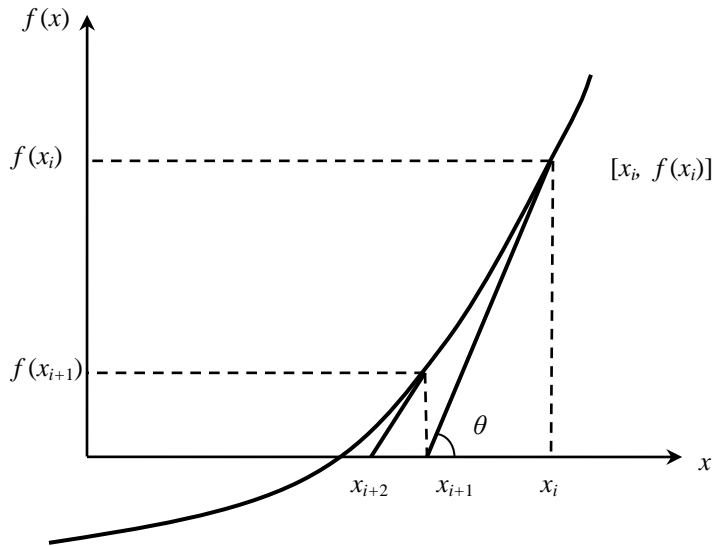
$$\begin{aligned} f'(x_i) &= \tan \theta \\ &= \frac{f(x_i) - 0}{x_i - x_{i+1}}, \end{aligned}$$

which gives

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

(1)

Equation (1) is called the Newton-Raphson formula for solving nonlinear equations of the form  $f(x) = 0$ . So starting with an initial guess,  $x_i$ , one can find the next guess,  $x_{i+1}$ , by using Equation (1). One can repeat this process until one finds the root within a desirable tolerance.



**Figure 1** Geometrical illustration of the Newton-Raphson method.