Algorithm for the bisection method

The steps to apply the bisection method to find the root of the equation $f(x) = 0$ are

1. Choose x_ℓ and x_u as two guesses for the root such that $f(x_\ell) f(x_u) < 0$, or in other words, $f(x)$ changes sign between x_ℓ and x_u.
2. Estimate the root, x_m, of the equation $f(x) = 0$ as the mid-point between x_ℓ and x_u as
$$x_m = \frac{x_\ell + x_u}{2}$$
3. Now check the following
   a) If $f(x_\ell) f(x_m) < 0$, then the root lies between x_ℓ and x_m; then $x_\ell = x_\ell$ and $x_u = x_m$.
   b) If $f(x_\ell) f(x_m) > 0$, then the root lies between x_m and x_u; then $x_\ell = x_m$ and $x_u = x_u$.
   c) If $f(x_\ell) f(x_m) = 0$; then the root is x_m. Stop the algorithm if this is true.
4. Find the new estimate of the root
$$x_m = \frac{x_\ell + x_u}{2}$$
Find the absolute relative approximate error as
$$\left| \in_a \right| = \left| \frac{x_m^{\text{new}} - x_m^{\text{old}}}{x_m^{\text{new}}} \right| \times 100$$
where
x_m^{new} = estimated root from present iteration
x_m^{old} = estimated root from previous iteration
5. Compare the absolute relative approximate error $\left| \in_a \right|$ with the pre-specified relative error tolerance $\in_s$. If $\left| \in_a \right| > \in_s$, then go to Step 3, else stop the algorithm. Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.