Example 2

The upward velocity of a rocket is given as a function of time in Table 2.

Table 1 Velocity as a function of time.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

Using a third order polynomial interpolant for velocity, find the acceleration of the rocket at $t = 16\,\text{s}$.

Solution

For the third order polynomial (also called cubic interpolation), we choose the velocity given by

$$v(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Since we want to find the velocity at $t = 16\,\text{s}$, and we are using a third order polynomial, we need to choose the four points closest to $t = 16$ and that also bracket $t = 16$ to evaluate it.

The four points are $t_0 = 10, t_1 = 15, t_2 = 20$ and $t_3 = 22.5$.

$$t_0 = 10,\ \ v(t_0) = 227.04$$
$$t_1 = 15,\ \ v(t_1) = 362.78$$
$$t_2 = 20,\ \ v(t_2) = 517.35$$
$$t_3 = 22.5,\ \ v(t_3) = 602.97$$

such that

$$v(10) = 227.04 = a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3$$
$$v(15) = 362.78 = a_0 + a_1(15) + a_2(15)^2 + a_3(15)^3$$
$$v(20) = 517.35 = a_0 + a_1(20) + a_2(20)^2 + a_3(20)^3$$
$$v(22.5) = 602.97 = a_0 + a_1(22.5) + a_2(22.5)^2 + a_3(22.5)^3$$

Writing the four equations in matrix form, we have

$$\begin{bmatrix} 1 & 10 & 100 & 1000 \\ 1 & 15 & 225 & 3375 \\ 1 & 20 & 400 & 8000 \\ 1 & 22.5 & 506.25 & 11391 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$$

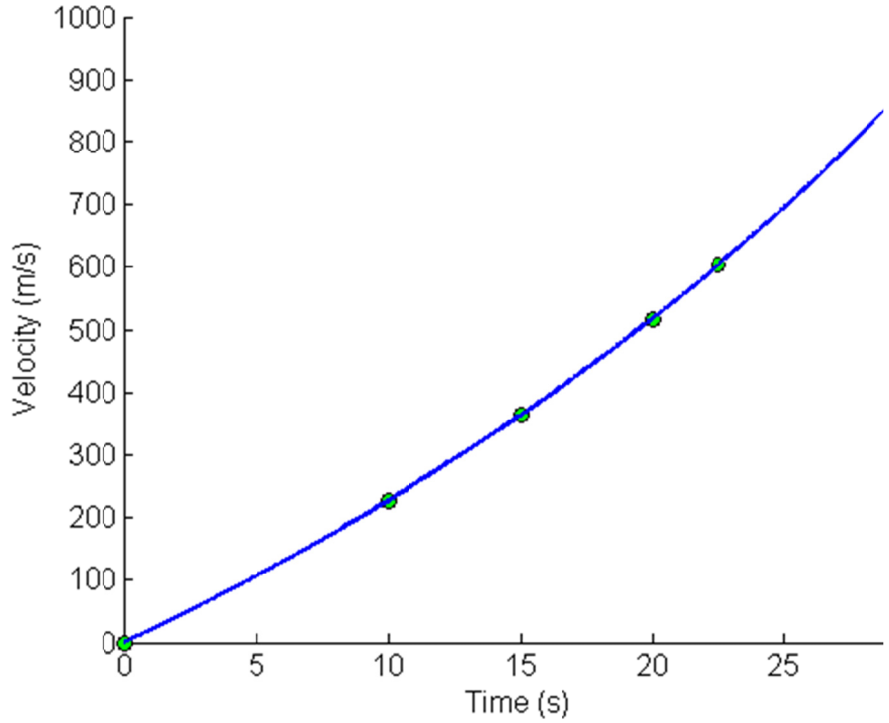

Figure 2 Graph of upward velocity of the rocket vs. time.

Solving the above four equations gives

$$a_0 = -4.3810$$
$$a_1 = 21.289$$
$$a_2 = 0.13065$$
$$a_3 = 0.0054606$$

Hence

$$v(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$= -4.3810 + 21.289t + 0.13065t^2 + 0.0054606t^3, \quad 10 \le t \le 22.5$$

The acceleration at $t = 16$ is given by

$$a(16) = \frac{d}{dt} v(t)\Big|_{t=16}$$

Given that $v(t) = -4.3810 + 21.289t + 0.13065t^2 + 0.0054606t^3, \quad 10 \le t \le 22.5$,

$$a(t) = \frac{d}{dt} v(t)$$
$$= \frac{d}{dt}\left(-4.3810 + 21.289t + 0.13065t^2 + 0.0054606t^3\right)$$
$$= 21.289 + 0.26130t + 0.016382t^2, \quad 10 \le t \le 22.5$$
$$a(16) = 21.289 + 0.26130(16) + 0.016382(16)^2$$
$$= 29.664 \text{ m/s}^2$$