**Example 5**

The velocity of a rocket is given by

$$\nu(t)=2000\ln\left[\frac{14\times10^4}{14\times10^4-2100t}\right]-9.8t,\ 0\le t\le 30,$$

(a) Use the central difference approximation of the second derivative of $\nu(t)$ to calculate the jerk at $t=16\,\text{s}$. Use a step size of $\Delta t=2\,\text{s}$.

**Solution**

The second derivative of velocity with respect to time is called jerk. The second order approximation of jerk then is

$$j(t_i)\approx\frac{\nu(t_{i+1})-2\nu(t_i)+\nu(t_{i-1})}{(\Delta t)^2}$$

$$t_i=16$$

$$\Delta t=2$$

$$\begin{aligned}t_{i+1}&=t_i+\Delta t\\&=16+2\\&=18\end{aligned}$$

$$\begin{aligned}t_{i+2}&=t_i-\Delta t\\&=16-2\\&=14\end{aligned}$$

$$j(16)\approx\frac{\nu(18)-2\nu(16)+\nu(14)}{(2)^2}$$

$$\begin{aligned}\nu(18)&=2000\ln\left[\frac{14\times10^4}{14\times10^4-2100(18)}\right]-9.8(18)\\&=453.02\,\text{m/s}\end{aligned}$$

$$\begin{aligned}\nu(16)&=2000\ln\left[\frac{14\times10^4}{14\times10^4-2100(16)}\right]-9.8(16)\\&=392.07\,\text{m/s}\end{aligned}$$

$$\begin{aligned}\nu(14)&=2000\ln\left[\frac{14\times10^4}{14\times10^4-2100(14)}\right]-9.8(14)\\&=334.24\,\text{m/s}\end{aligned}$$

$$\begin{aligned}j(16)&\approx\frac{\nu(18)-2\nu(16)+\nu(14)}{(2)^2}\\&=\frac{453.02-2(392.07)+334.24}{4}\\&=0.77969\,\text{m/s}^3\end{aligned}$$

The absolute relative true error is

$$\begin{aligned}|\in_t|&=\left|\frac{0.77908-0.77969}{0.77908}\right|\times100\\&=0.077992\%\end{aligned}$$