

Forward Difference Approximation from Taylor Series

Taylor's theorem says that if you know the value of a function $f(x)$ at a point x_i and all its derivatives at that point, provided the derivatives are continuous between x_i and x_{i+1} , then

$$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + \frac{f''(x_i)}{2!}(x_{i+1} - x_i)^2 + \dots$$

Substituting for convenience $\Delta x = x_{i+1} - x_i$

$$f(x_{i+1}) = f(x_i) + f'(x_i)\Delta x + \frac{f''(x_i)}{2!}(\Delta x)^2 + \dots$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} - \frac{f''(x_i)}{2!}(\Delta x) + \dots$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{\Delta x} + O(\Delta x)$$

The $O(\Delta x)$ term shows that the error in the approximation is of the order of Δx .

Can you now derive from the Taylor series the formula for the backward divided difference approximation of the first derivative?