Example 2

The velocity of a rocket is given by

$$v(t) = 2000\ln\left[\frac{14\times10^4}{14\times10^4 - 2100t}\right] - 9.8t, 0 \le t \le 30$$

(a) Use the backward difference approximation of the first derivative of $v(t)$ to calculate the acceleration at $t = 16\text{s}$. Use a step size of $\Delta t = 2\text{ s}$.

(b) Find the absolute relative true error for part (a).

Solution

$$a(t) \approx \frac{v(t_i) - v(t_{i-1})}{\Delta t}$$

$$t_i = 16$$

$$\Delta t = 2$$

$$t_{i-1} = t_i - \Delta t$$

$$= 16 - 2$$

$$= 14$$

$$a(16) \approx \frac{v(16) - v(14)}{2}$$

$$v(16) = 2000\ln\left[\frac{14\times10^4}{14\times10^4 - 2100(16)}\right] - 9.8(16)$$

$$= 392.07\text{ m/s}$$

$$v(14) = 2000\ln\left[\frac{14\times10^4}{14\times10^4 - 2100(14)}\right] - 9.8(14)$$

$$= 334.24\text{ m/s}$$

$$a(16) \approx \frac{v(16) - v(14)}{2}$$

$$= \frac{392.07 - 334.24}{2}$$

$$= 28.915\text{ m/s}^2$$

(b) The exact value of the acceleration at $t = 16\text{s}$ from Example 1 is

$$a(16) = 29.674\text{ m/s}^2$$

The absolute relative true error for the answer in part (a) is

$$|\in_t| = \left|\frac{29.674 - 28.915}{29.674}\right| \times 100$$

$$= 2.5584\%$$